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***Trace Anomaly and Dimension Two Gluon Condensate Above  
the Phase Transition***

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# Trace Anomaly and Dimension Two Gluon Condensate Above the Phase Transition\*

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The dimension two gluon condensate has been used previously within a simple phenomenological model to describe power corrections from available lattice data for the renormalized Polyakov loop and the heavy quark-antiquark free energy in the deconfined phase of QCD [1, 2]. The QCD trace anomaly of gluodynamics also shows unequivocal inverse temperature power corrections which may be encoded as dimension two gluon condensate. We analyze lattice data of the trace anomaly and compare with other determinations of the condensate from previous references, yielding roughly similar numerical values.

**Introduction.** For zero and for infinite quark masses (gluodynamics) QCD is invariant under scale and conformal transformations at the classical level. This classical invariance is broken, however, by quantum corrections due to the necessary regularization of ultraviolet divergences which introduces a mass scale,  $\Lambda_{\text{QCD}}$ ; the divergence of the dilatation current equals the trace of the improved energy-momentum tensor  $\Theta^\mu_\mu$  [3] yielding the so-called “trace anomaly” [4]. At finite temperature, the energy density  $\epsilon$  and the pressure  $p$  enter as [5, 6, 7, 8],

$$\epsilon - 3p = \frac{\beta(g)}{2g} \langle (G_{\mu\nu}^a)^2 \rangle \equiv \langle \Theta^\mu_\mu \rangle, \quad (1)$$

where  $G_{\mu\nu} = \partial_\mu A_\nu - \partial_\nu A_\mu + ig[A_\mu, A_\nu]$  is the field strength tensor and  $\beta(g) = \mu \partial g / \partial \mu$  is the beta function. Far from the conformal limit, where  $\epsilon = 3p$ ,  $\Delta = (\epsilon - 3p)/T^4$  is a dimensionless quantity providing a measure of the interaction, so it is commonly known as “interaction measure”. A good knowledge of  $\Delta$  is crucial to understand the deconfinement process, where the non perturbative (NP) nature of low energy QCD seems to play a prominent role. In this contribution we analyze the highly NP behaviour of the trace anomaly just above the phase transition and describe it in a way that is consistent with other thermal observables (see [9] for further details).

**Thermal power corrections in gluodynamics.** The interaction measure was computed one decade ago on the lattice by the Bielefeld group for gluodynamics [10]. Fig. 1 shows the lattice data for  $\Delta = (\epsilon - 3p)/T^4$  as a function of  $T/T_c$ .  $\Delta$  is very small below  $T_c$ , because the lightest glueball is much heavier than  $T_c \approx 270$  MeV. It increases suddenly near and above  $T_c$  by latent heat of deconfinement, and raises a maximum at  $T \approx 1.1 T_c$ . Then it has a gradual decrease reaching zero in the high temperature limit. The high value of  $\Delta$  for  $T_c \lesssim T \lesssim (2.5 - 3)T_c$  corresponds to a strongly interacting Quark-Gluon Plasma picture.

From our previous experience [1, 2] and following a remark by Pisarski [11], in Fig. 1 we plot  $(\epsilon - 3p)/T^4$  as a function of  $1/T^2$  (in units of  $T_c$ ) exhibiting an unmistakable straight line behaviour

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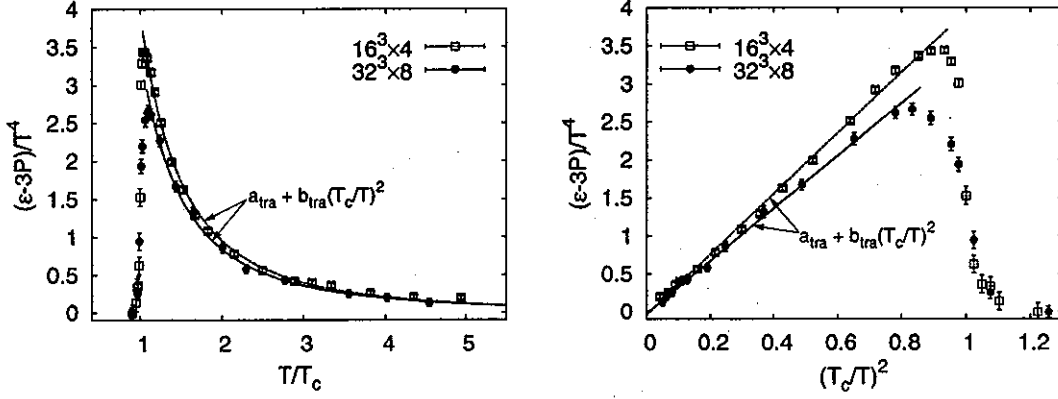


FIG. 1: The trace anomaly density  $(\epsilon - 3P)/T^4$  as a function of  $T$  (left) and  $1/T^2$  (right) (in units of  $T_c$ ). Lattice data are from [10] for  $N_\sigma^3 \times N_\tau = 16^3 \times 4$  and  $32^3 \times 8$ . The fits use Eq. (2) with  $a_{\text{tra}}$  and  $b_{\text{tra}}$  adjustable constants.

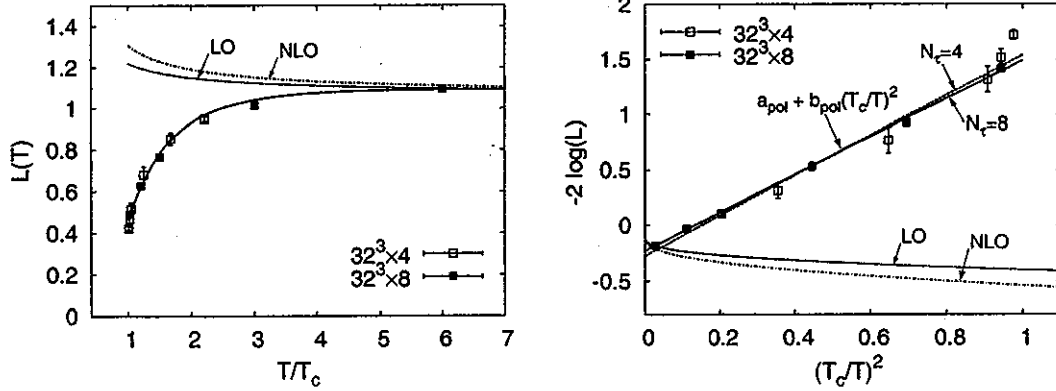


FIG. 2: The temperature dependence of the renormalized Polyakov loop in units of the critical temperature. Lattice data are from [14], for  $N_\sigma^3 \times N_\tau = 32^3 \times 4$  and  $32^3 \times 8$ . We plot the perturbative result at LO and NLO, and the fit using  $a_{\text{pol}} + b_{\text{pol}}(T_c/T)^2$ .

in the region slightly above the critical temperature, of the form

$$\Delta = (\epsilon - 3P)/T^4 = a_{\text{tra}} + b_{\text{tra}}(T_c/T)^2, \quad (2)$$

and corresponding to a “power correction” in temperature. A fit of the lattice data ( $N_\sigma^3 \times N_\tau = 32^3 \times 8$ ) for  $T/T_c > 1.13$  yields  $a_{\text{tra}} = -0.02(4)$ ,  $b_{\text{tra}} = 3.46(13)$ ,  $\chi^2/\text{DOF} = 0.35$ . Power corrections also appear in  $\epsilon$  and  $P$ , just by applying the standard thermodynamic relations.

This behaviour clearly contradicts perturbation theory (PT) which contains no powers but only logarithms in the temperature, a feature shared by hard thermal loops and other resummation techniques (see e.g. [12, 13]), explaining why they have failed to describe lattice data of the free energy below  $3T_c$ . Fig. 2 shows the lattice data of Polyakov loop from Ref. [14], suggesting again [1, 2] a linear fit of the form  $-2 \log L = a_{\text{pol}} + b_{\text{pol}}(T_c/T)^2$ . In what follows, we show a phenomenological model that describes consistently all these power corrections in an unified way.

**Dimension two gluon condensate.** The gluon condensate  $\langle G^2 \rangle \equiv g^2 \langle (G_{\mu\nu}^a)^2 \rangle$  describes the anomalous (and not spontaneous) breaking of scale invariance, and hence is not an order

parameter of the phase transition. Actually, the order parameter is the vacuum expectation value of the Polyakov loop which signals the breaking of the  $Z(N_c)$  discrete symmetry of gluodynamics as well as the deconfinement transition. A dimension two gluon condensate naturally appears from a computation of the Polyakov loop in which a Gaussian distribution of eigenvalues is considered. In the static gauge,  $\partial_0 A_0(\mathbf{x}, x_0) = 0$ , this Gaussian-like, large  $N_c$  motivated, approximation gives [1]

$$L(T) = \left\langle \frac{1}{N_c} \text{tr}_c e^{igA_0(\mathbf{x})/T} \right\rangle = \exp \left[ -\frac{g^2 \langle A_{0,a}^2 \rangle}{4N_c T^2} \right] + \mathcal{O}(g^6), \quad (3)$$

valid up to  $\mathcal{O}(g^5)$  in PT.  $A_0$  is the gluon field in the (Euclidean) time direction. From here it is immediate to relate the Polyakov loop to the gluon propagator in the dimensionally reduced theory

$$\delta_{ab} T D_{00}(\mathbf{k}) = \int d^3 \mathbf{x} \langle A_{0,a}(\mathbf{x}) A_{0,b}(\mathbf{y}) \rangle e^{-i\mathbf{k} \cdot (\mathbf{x} - \mathbf{y})}. \quad (4)$$

The dimension two gluon condensate  $g^2 \langle A_{0,a}^2 \rangle$  is obtained from Eq. (4) in the limit  $\mathbf{x} \rightarrow \mathbf{y}$ . The perturbative propagator  $D_{00}^P(\mathbf{k}) = 1/(\mathbf{k}^2 + m_D^2) + \mathcal{O}(g^2)$ , being  $m_D \sim T$  the Debye mass, leads to the known perturbative result of Gava and Jengo [15], which fails to reproduce lattice data below  $6T_c$ . A NP model is proposed in Ref. [1] to describe the lattice data of the Polyakov loop, and it consists in a new piece in the gluon propagator driven by a positive mass dimension parameter:

$$D_{00}(\mathbf{k}) = D_{00}^P(\mathbf{k}) + D_{00}^{\text{NP}}(\mathbf{k}), \quad D_{00}^{\text{NP}}(\mathbf{k}) = m_G^2/(\mathbf{k}^2 + m_D^2)^2. \quad (5)$$

This ansatz parallels a zero temperature one [16], where the dimension two condensate provides the short-distance NP physics of QCD and at zero temperature this contribution yields the well known NP linear term in the  $\bar{q}q$  potential. A justification of Eq. (5) based on Schwinger-Dyson methods has been given [17]. The new propagator generates a NP contribution to the condensate,  $\langle A_{0,a}^2 \rangle = \langle A_{0,a}^2 \rangle^P + \langle A_{0,a}^2 \rangle^{\text{NP}}$ , which is related to  $m_G^2$  through  $\langle A_{0,a}^2 \rangle^{\text{NP}} = (N_c^2 - 1)m_G^2/(8\pi\hat{m}_D)$ , where  $\hat{m}_D \equiv m_D/T$ , so that it leads to the thermal power behaviour that we observe in Fig. 2. The Gaussian approximation has also been used in Ref. [2] to compute the singlet free energy of a heavy  $\bar{q}q$  pair [14, 18], through the correlation function of Polyakov loops.

**Non perturbative contribution to the Trace Anomaly.** The model of Eq. (5) can be easily used to compute the trace anomaly Eq. (1) in gluodynamics. Assuming the leading NP contribution to be encoded in the  $A_{0,a}$  field and taking  $A_{i,a} = 0$  yields

$$\langle G_{\mu\nu}^a G_{\mu\nu}^a \rangle^{\text{NP}} = 2 \langle \partial_i A_{0,a} \partial_i A_{0,a} \rangle^{\text{NP}} = -6m_D^2 \langle A_{0,a} A_{0,a} \rangle^{\text{NP}}. \quad (6)$$

The r.h.s. is obtained from Eq. (4) by expanding in the limit  $\mathbf{x} \rightarrow \mathbf{y}$  and looking at the quadratic term in  $r = |\mathbf{x} - \mathbf{y}|$ . Note that the NP model is formulated in the dimensionally reduced theory, so the gluon fields are static. This formula produces the thermal power behaviour of Eq. (2) with

$$b_{\text{tra}} T_c^2 = -3\hat{m}_D^2 \langle A_{0,a}^2 \rangle^{\text{NP}} \beta(g)/g. \quad (7)$$

If we consider the perturbative value of the beta function  $\beta(g) \sim g^3 + \mathcal{O}(g^5)$ , the r.h.s. of Eq. (7) shows a factor  $g^2$  in addition to the dimension two gluon condensate  $g^2 \langle A_{0,a}^2 \rangle^{\text{NP}}$ . So the fit of the trace anomaly data is sensitive to the value of the smooth  $T$ -dependent  $g$ , without jeopardizing the power correction. For the Polyakov loop the sensitivity in  $g$  is only through the perturbative terms, which are much smaller than the NP ones. When we consider the perturbative value  $g_P$  up to 2-loops, we get from the fit of the trace anomaly  $g^2 \langle A_{0,a}^2 \rangle^{\text{NP}} = (2.63 \pm 0.05 T_c)^2$ , which is a factor 1.5 smaller than what is obtained from other observables. This disagreement could be partly explained on the basis of certain ambiguity of  $g$  in the NP regime. A better fit of the Polyakov loop and heavy quark free energy lattice data in the regime  $T_c < T < 4T_c$  is obtained for a slightly smaller  $g$  than

Observable	$g^2 \langle A_{0,a}^2 \rangle^{NP}$
Polyakov loop [1]	$(3.22 \pm 0.07 T_c)^2$
Heavy $\bar{q}q$ free energy [2]	$(3.33 \pm 0.19 T_c)^2$
Trace Anomaly	$(2.86 \pm 0.24 T_c)^2$

TABLE I: Values of the dimension two gluon condensate from a fit of several observables in the deconfined phase of gluodynamics: Polyakov loop, singlet free energy of heavy quark-antiquark and trace anomaly. Values are in units of  $T_c$ . We show the fit for lattice data with  $N_\tau = 8$ . Error in last line takes into account an indeterminate value of the coupling constant  $g = 1.26 - 1.46$ , being the highest value the perturbative  $g_P$  up to 2-loops at  $T = 2T_c$ . The critical temperature in gluodynamics is  $T_c = 270 \pm 2$  MeV [14].

$g_P$ , i.e.  $g = 1.26 - 1.46$  [2]. Taking this value we get from Eq. (7)  $g^2 \langle A_{0,a}^2 \rangle^{NP} = (2.86 \pm 0.24 T_c)^2$ , in better agreement with other determinations, see Table I. Nonetheless, an alternative method to compute the trace anomaly based on the direct computation of  $\epsilon - 3p$  from the partition function of gluodynamics does reproduce the power correction, however with different coefficients [9].

**Summary and conclusions.** The trace anomaly in gluodynamics shows, near and above the critical temperature, a clear pattern of power corrections which cannot be matched to perturbation theory or resummations thereof. It can instead be explained in terms of a dimension two gluon condensate whose numerical value agrees with other determinations based on other thermal observables and it is also remarkably close to existing studies at  $T = 0$  (see e.g. Refs. [19, 20]).

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